QUANTITATIVE INDEX BOUNDS FOR WEIGHTED MINIMAL HYPERSURFACES VIA TOPOLOGY

MICHELE RIMOLDI

Dipartimento di Scienze Matematiche
Politecnico di Torino

Abstract. The recent impressive developments in the existence theory for minimal immersions have motivated a renewed interest in studying estimates on the Morse index of these objects. One possible way to control instability is through topological invariants (in particular the first Betti number) of the minimal hypersurface. This was first investigate by A. Ros for immersed minimal surfaces in $\mathbb{R}^3$, or a quotient of it by a group of translations, and then, in higher dimension, by A. Savo when then ambient manifold is a round sphere. In this talk we will first discuss how the method of Savo can be generalized to study the Morse index of self-shrinkers for the mean curvature flow and, more generally, of weighted minimal hypersurfaces in a weighted Euclidean space endowed with a convex weight. When the hypersurface is compact, we will show that the index is bounded from below by an affine function of its first Betti number. When the first Betti number is large this improves index estimates known in literature. In the complete non-compact case, the lower bound is in terms of the dimension of the space of weighted square integrable $f$-harmonic 1-forms. In particular, in dimension 2, the procedure gives an index estimate in terms of the genus of the surface.

Combining this technique with an adaptation to the weighted setting of Li-Tam theory we will also discuss some recent quantitative estimates on the Morse index of translators for the mean curvature flow with bounded norm of the second fundamental form via the number of ends of the hypersurface.

This talk is based on joint works with Debora Impera and Alessandro Savo.