Streak speckle velocimetry

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We present a method for fluid velocimetry based on a single-exposure analysis of the streak speckle pattern generated by sub-micron tracking particles illuminated with coherent light. It works in real-time and provides two dimensional velocity mappings in the direction orthogonal to the optical axis, independently of particle concentration and size. It is immune of any spurious light acting as undesired heterodyne signal and can probe velocities much higher (~three orders of magnitude) than methods based on double-exposure analysis. The method has been tested by using rigid diffusers of different heterodyne strength and applied to map the flow of a confined fluid.

The characterization of velocity mappings of fluid flows in confined three dimensional (3D) domains is one of the main concerns of modern fluid dynamics. Many are the velocimetry techniques that have been developed over the years for undertaking this difficult task. They work by seeding the fluid with small tracking particles, illuminating the sample with a luminous source and detecting the light scattered by the particles with a two dimensional (2D) sensor. Velocimetry techniques can be grouped in a few main categories, depending on a double- or single-exposure of the scattered light and on the capability of resolving or not-resolving single particles.

Probably, the two most popular (and simple) methods belonging to the first category (double-exposure) are Particle Imaging Velocimetry (PIV) and Laser Speckle Velocimetry (LSV). In their simplest realizations, PIV and LSV work by illuminating the seeded fluid with a thin sheet of laser light and imaging the particles along a perpendicular direction. When the numerical aperture (NA) of the collection optics is high enough and its depth of focus is longer than the sample thickness, as well as the tracers are not too concentrated, single particles can be resolved (PIV), otherwise they cannot. In the latter case, due to the coherent illumination, the detected image assumes a speckled appearance (LSV). The data analysis of PIV and LSV is identical: by cross-correlating two images acquired at two closely spaced times, one can determine the local tracers displacements and consequently recover the 2D in-plane velocity mapping of the fluid. 3D implementations of these techniques are possible, such as Stereo PIV (SPIV) or Holographic PIV (HPIV), but the setups and the overall procedures may become rather complex.

Velocimetry techniques based on the second category (single-exposure) are less popular than the first ones, but their use has been constantly increasing over the last decade. They work by integrating the scattered light over a long exposure time $t_{exp}$, so that, when the optics is capable of resolving single particles, the image of a moving particle appears as a streak whose length is related to particle displacement. In this case, the technique is known with the name of Particle Streak Velocimetry (PSV). Conversely, when the particles are not resolved, a laser-light illumination of the sample produces a streak speckled image, whose contrast becomes increasingly smaller as $t_{exp}$ is made longer than the speckle pattern decorrelation time. Typically, the analysis of these images is based only on the measurement of their contrast, either in the time-domain (Laser Speckle Imaging (LSI)) or in the spatial-domain (Laser Speckle Contrast Analysis (LASCA)). These techniques suffer of being poorly quantitative and do not provide any info on velocity directions, but have the great advantage of being relatively immune from multiple scattering, so that they find beautiful applications in the field of biomedical imaging (such as for visualizing blood tissue perfusion).

In this Letter, we propose a single-exposure, streak speckle velocimetry (SSV) technique based on the entire morphological analysis of the speckle pattern correlation function. Indeed, due to particle motion, the speckle pattern is not only blurred (low contrast) but also characterized by a triangular correlation function (see below) whose Full Width Half Maximum (FWHM) is closely related to the distance traveled by the particles during $t_{exp}$. The method provides real-time velocity mappings of a fluid moving in a plane, can work with tracers below the resolution of the optics, relatively high concentrations can be used, and is quite robust against the presence of any undesired heterodyne signal. Finally, being based on a single-exposure analysis of the detected image, no fast expensive 2D detectors are needed.

SSV requires a simple optical setup (see Fig. 1), similar to the one used in Heterodyne Near Field Scattering (HNFS) or Heterodyne Speckle Velocimetry (HSV), in which a large collimated laser beam of diameter $D$ and wavelength $\lambda$ is sent onto a square cell containing a moving fluid seeded with small tracking particles of diameter $d$. By using an optical microscope objective and a CCD camera, the light falling onto a plane located at a close distance $z$ from the sample can be easily recorded. Here, the intensity distribution is the result of the interference between the transmitted and the light scattered by the tracers, i.e.,

$$I_0(x,y) = |E_T(x,y) + E_s(x,y)|^2,$$

where $E_T$ and $E_s$ are the transmitted and scattered fields, respectively. If the particles are sufficiently small ($d \leq \lambda$), the maximum scattering angle collected by the optics is $\theta_{max} \sim NA$ (NA being the numerical aperture of the microscope) and in turn, the size $\delta_0$ of the (subjective) speckles is $\delta_0 \sim \lambda/(2NA)$. If the distance $z$ of the observation plane is sufficiently small, the transversal region
from which the scattered light is collected, $D^\ast \sim 2 \theta_{\text{max}}$, is smaller than the beam size $D$. When this condition is strongly fulfilled ($D^\ast \ll D$), we are in the so called **deep Fresnel** region,\textsuperscript{19–21} where the speckles formed around any given point $r$ of the observation plane are determined only by those particles lying within the region $D^\ast$ centered on the back projection of $r$ over the scattering cell. Thus, if the sample is moved transversally to the optical axis, the speckles move accordingly, and by integrating $I_0(x,y)$ over an exposure time $t_{\text{exp}}$ long enough that the detected speckle pattern is streaked along the particle motion, one can recover the distance traveled by the fluid during $t_{\text{exp}}$.

The theory behind SSV is fairly simple: let us suppose, for the sake of simplicity, that the particles move along the $x$-direction at a constant velocity $v$. Thus, the time-averaged intensity reads

$$I(x,y) = \frac{1}{t_{\text{exp}}} \int_{-t_{\text{exp}}}^{t_{\text{exp}}} I_0(x - vt',y) \, dt', \quad (1)$$

where $I_0(x,y)$ is the instantaneous intensity distribution corresponding to still particles ($v=0$). Equation (1) can be re-written in terms of a spatial average as

$$I(x,y) = \frac{1}{L} \int_{-\infty}^{\infty} I_0(x-L/2 - x',y) \, R(x'/L) \, dx', \quad (2)$$

where $L = v \, t_{\text{exp}}$ and the $R(x)$ is the rectangle function defined as $R(x)=1$ for $|x| \leq 0.5$ and 0 elsewhere. Note that, as expressed by the second line of Eq. (2), $I(x,y)$ is given by the one dimensional (1D) convolution (symbol $\otimes$) between $R(x)$ and $I_0(x,y)$ (shifted by $L/2$). The (normalized) autocorrelation function of $I(x,y)$, i.e.,

$$g_0(\xi,y) = \int I(x,y)I(\xi + x,y + \eta)/|I(x,y)|^2 \, dx, \quad (6)$$

is given by

$$g_0(\xi,y) = \int g_0(0,y) \frac{\Lambda(\xi/L)}{L} \, dx, \quad (4)$$

where $g_0(\rho)$ is the azimuthal averaged radial profile of $g_0(\xi,y)$, being $\rho = \sqrt{\xi^2 + \eta^2}$. Differently, for the orthogonal component, we have

$$g_\perp(\eta) = \int g_0(0,\eta) \frac{\Lambda(\xi/L)}{L} \, dy.$$
The figure shows that, over the entire range of velocities versus actual velocities for the three diffusers even when $\delta g/\delta d$ is much broader than $\delta g/\delta K$. Therefore, convolution effects are negligible (see Eq. (4)) and the shape of $g_\|/g_\parallel$ is very close to that of an ideal triangle. However, when the widths of $g_\parallel$ and $g_\perp$ are comparable, the accurate recovery of the triangle and of its width $\delta g$ would required the deconvolution of Eq. (4), a non-trivial task that is beyond the purpose of this work.

Here, we propose an empirical method for the recovery of $\delta g$ based on the quadratic difference between the FWHM of $g_\parallel$ and $g_\perp$, which reads

$$\delta g \simeq \sqrt{(\delta_\parallel)^2 - (\delta_\perp)^2} \tag{7}$$

from which one recovers the particles velocity as $v = \delta g/\tau_{exp}$. Equation (7) is highly accurate when $\delta_\parallel \gg \delta_\perp$, but even when $\delta_\parallel/\delta_\perp \geq 5$ provides results that are accurate within $\pm 5\%$, as it can be easily shown numerically.

In Fig. 4(a), we report the behaviors of the recovered velocities versus actual velocities for the three diffusers described above. The figure shows that, over the entire investigated range ($\sim 3$ decades), the recovered velocities match quite accurately the expected ones, as also shown in the residual plot (Fig. 4(b)) with nonsystematic r.m.s. deviations of $\sim 3\%$. The different symbols refer to exposure times that were varied in a range between 600 ms and 1 ms, so that $\delta g/\delta K \geq 5$. It should be pointed out that the highest velocities ($\sim 300$ mm/s) reported in the figure were not limited by the technique, but by the maximum angular speed achievable with our rotating diffusers. Indeed, the maximum velocity retrievable with the SSV technique should be $v_{max} \sim A/(\tau_{exp})_{min}$, where $A$ denotes the effective sensor size and $(\tau_{exp})_{min}$ is the minimum exposure time of the detector. In our case, $A \sim 1$ mm and $(\tau_{exp})_{min} \sim 0.1$ ms, and therefore $v_{max} \sim 10$ m/s. Notice that these velocities much higher that the ones achievable with double-exposure methods, because in that case, $v_{max} \sim A/T$, ($T$ being the period between two images) and, typically, $T/(\tau_{exp})_{min} \sim 10^{3}$.

Finally, we show how the SSV technique works on a real flowing fluid made of a water suspension of latex particles, 700 nm in diameter, fluxed at $3 \times 10^{-3}$ ml/s inside a square $\sim 1.5$ mm thick cell whose width was tapered as a funnel, similarly to what done in Refs. 17 and 18. The setup was the same as in Fig. 1, but with $z = 0$. A typical frame acquired at $\tau_{exp} = 3$ ms is shown in Fig. 5(a), where the streak speckles appear to be nicely oriented and elongated along the channel. The analysis, carried out on an array of $16 \times 16$ subframes ($100 \times 100$ $\mu m^2$), is shown in Fig. 5(b), where the segments indicate direction and modulus of the various velocities. As expected for an incompressible viscous fluid undergoing a laminar flux, the velocity increases both in the middle of the channel and as the funnel neck gets narrower. Typical figures range between $v \sim 3-15$ mm/s with an average value $\langle v \rangle \sim 8 \pm 3$ mm/s. As described in Refs. 17 and 18, diffusion does not play any role in tracers’ motion. The detailed analysis of each subframe (insets of Fig. 5(b), data not shown) reveals that the shape of $g_\parallel(\xi)$ is significantly different from that of a triangle (similar to a witch’s hat), which is a clear indication of the presence of a velocity distribution.

![FIG. 2. Typical image (a) and autocorrelation function (b) recovered with the streak speckle velocimetry technique for a glycerine diffuser. The dashed yellow and red lines of panel (b) represent the sections of the correlation functions parallel and orthogonal to particles velocity, respectively.](image1)

![FIG. 3. Parallel $g_\parallel$ (blue) and orthogonal $g_\perp$ (red) sections of the streak autocorrelation function reported in panel (b) of Fig. 2.](image2)

![FIG. 4. Recovered velocities versus sample velocities (a) and corresponding relative residuals (b) for three index matched rotating diffusers, made by sticking together a ground and a transparent glass with in between air (red symbols), glycerine (blue symbols), and cedar tree oil (green symbols). The lines trough the symbols represent the ideal behavior $v_{recov} = v_{sample}$. For the sake of clarity, blue and green data (and lines) have been shifted upwards in panel (a).](image3)
inside that region. A quantitative analysis of this effect in terms of the Poiseuille flow is deferred to future work.

In conclusion, we have proposed a simple method for 2D real-time velocimetry of fluids seeded with small tracking particles, based on a single-exposure analysis of the streak speckle pattern generated by the particles. Being based on the analysis of the shape (not of the amplitude) of the autocorrelation function, the method works accurately under any heterodyning condition, i.e., independently of the ratio $I_T/I_S$. The latter feature makes the SSV technique much more powerful than methods based on the measure of the speckle pattern contrast only, such as LASCA and LSI. However, whereas LASCA and LSI work reasonably well even when the moving fluid is immersed in a moderate turbid medium (like a tissue) and therefore can be used for biomedical imaging applications,\textsuperscript{12–14} in the case of SSV, the situation is more critical. SSV requires single scattering conditions and, consequently, can map velocities of a fluid moving in a quasi-transparent environment (either in a transmission or reflection geometry), but not in a turbid medium.

When compared with methods based on a double-exposure analysis of the speckle pattern (such as PIV, LSV, or HSV), SSV allows the recovery of much higher velocities ($\sim$three orders of magnitude) because for a standard slow CCD’s, the ratio between the minimum exposure time and minimum frame period is $\sim10^{-3}$. The drawback is that, whereas in PIV, LSV, or HSV, the minimum detectable particle displacement corresponds approximately to the speckle size $\delta_0$, in SSV the minimum distance is $\sim5\delta_0$, a limit that can be overcome if, instead of Eq. (7), one would deconvolve Eq. (3). As a final comment, we would like to point out that, similarly to what happens in HSV,\textsuperscript{17,18} SSV requires a pure transversal motion of the tracers and only average velocities (averaged over the sample thickness) can be retrieved. However, when the method is operated in the strong heterodyne regime ($I_T/I_S \rightarrow \infty$), the measured autocorrelation function becomes linear in the number of tracers,\textsuperscript{17,18} and it should be possible to retrieve the velocity distribution along the sample thickness. Work is in progress to achieve this goal.

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